## Math 1B

## Midterm 3 Review

For sections 5.4–5.5 and 7.1–7.5:

Get together a group of classmates.

Make a copy of the following pages and integrals:

Cut them up, so each question is on a separate slip of paper.

Throw them in a pile and mix them up.

One at a time, randomly pick out a question from the pile and everyone solve it independently.

Compare solutions and discuss which one is fastest/easiest, and how you can recognize what method to use.

The following questions act as a review for 7.8.

[1] Determine if the following integrals converge or diverge. If an integral converges, find its value.

$$[a] \qquad \int\limits_0^\infty x^2 e^{-3x} dx$$

[b] 
$$\int_{0}^{\infty} \frac{1}{\sqrt[3]{x-1}} dx$$

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + 4} \, dx$$

$$[d] \qquad \int_{-\infty}^{\infty} \frac{x}{x^2 + 4} \, dx$$

[e] 
$$\int_{-\infty}^{0} \frac{e^x}{1 + e^x} dx$$

[f] 
$$\int_{0}^{2} \frac{1}{\sqrt{4-x^2}} dx$$

[a] 
$$\int_{0}^{\infty} x^{2} e^{-3x} dx$$
 [b] 
$$\int_{0}^{\infty} \frac{1}{\sqrt[3]{x-1}} dx$$
 [c] 
$$\int_{-\infty}^{\infty} \frac{1}{x^{2}+4} dx$$
 [d] 
$$\int_{-\infty}^{\infty} \frac{x}{x^{2}+4} dx$$
 [e] 
$$\int_{-\infty}^{0} \frac{e^{x}}{1+e^{x}} dx$$
 [f] 
$$\int_{0}^{2} \frac{1}{\sqrt{4-x^{2}}} dx$$
 [g] 
$$\int_{0}^{2} \frac{x}{\sqrt{4-x^{2}}} dx$$
 [h] 
$$\int_{0}^{1} \frac{1}{x(\ln x)^{2}} dx$$

$$[h] \qquad \int_0^1 \frac{1}{x(\ln x)^2} dx$$

[i] 
$$\int_{0}^{\pi} \tan x \, dx$$

[2] Determine if the following integrals converge or diverge. Justify your answer.

[a] 
$$\int_{1}^{\infty} \frac{2 + \sin x}{x} dx$$

$$[b] \qquad \int_{1}^{\infty} \frac{2 + \sin x}{x^2} dx$$

$$c] \qquad \int_{0}^{\infty} e^{-x^2} dx$$

[d] 
$$\int_{e}^{\infty} \frac{1}{\ln x} dx$$

[e] 
$$\int_{e}^{\infty} \frac{1}{x \ln x} dx$$

$$[f] \qquad \int\limits_{2}^{\infty} \frac{x+1}{\sqrt{x^4-1}} \, dx$$

$$\int_{1}^{\infty} \frac{2 + \sin x}{x} dx \qquad [b] \qquad \int_{1}^{\infty} \frac{2 + \sin x}{x^{2}} dx \qquad [c] \qquad \int_{0}^{\infty} e^{-x^{2}} dx$$

$$\int_{e}^{\infty} \frac{1}{x \ln x} dx \qquad [f] \qquad \int_{2}^{\infty} \frac{x + 1}{\sqrt{x^{4} - 1}} dx \qquad [g] \qquad \int_{1}^{\infty} \frac{\cos^{2} x}{x e^{x}} dx$$

Answers

[1] [a] 
$$\frac{2}{27}$$

[c] 
$$\frac{\pi}{2}$$

[f] 
$$\frac{\pi}{2}$$

[i] diverges

[2] [a] diverges – compare to 
$$\frac{1}{x}$$

[b] converges – compare to 
$$\frac{3}{x^2}$$

[c] converges – compare to 
$$e^{-x}$$

[d] diverges – compare to 
$$\frac{1}{x}$$

[f] diverges – compare to 
$$\frac{1}{x}$$

[g] converges – compare to 
$$\frac{1}{e^x}$$